Archdiocese of Washington Catholic Schools
Academic Standards
Mathematics

ALGEBRA 1

Standard 1 – Operations with Real Numbers

Students simplify and compare expressions. They use rational exponents, and simplify square roots.

A1.1.1   Compare real number expressions.

Example: Compare the sizes of: $\sqrt{36} + \sqrt{64}$ and $\sqrt{36+64}$.

A1.1.2   Simplify square roots using factors.

Example: Explain why $\sqrt{48} = 4\sqrt{3}$

A1.1.3   Understand and use the distributive, associative, and commutative properties.

Example: Simplify $(6x^2 - 5x + 1) - 2(x^2 + 3x - 4)$ by removing the parentheses and rearranging. Explain why you can carry out each step.

A1.1.4   Use the laws of exponents for rational numbers.

Example: Simplify $25^{3/2}$.

A1.1.5   Use dimensional (unit) analysis to organize conversions and computations.

Example: Convert 5 miles per hour to feet per second.

Standard 2 – Linear Equations and Inequalities

Students solve linear equations and inequalities in one variable. They solve word problems that involve linear equations, inequalities, or formulas.

A1.2.1   Solve linear equations.

Example: Solve the equation $7a + 2 = 5a - 3a + 8$.

A1.2.2   Solve equations and formulas for a specified variable.

Example: Solve the equation $q = 4p - 11$ for p.

A1.2.3   Find solution sets of linear inequalities when possible numbers are given for a variable.

Example: Solve the inequality $6x - 3 > 10$ for x in the set $\{0, 1, 2, 3, 4\}$.

A1.2.4   Solve linear inequalities using properties of order.

Example: Solve the inequality $8x - 7 \leq 2x + 5$, explaining each step in your solution.

A1.2.5   Solve combined linear inequalities.

Example: Solve the inequalities $-7 > 3x + 5 > 11$.

A1.2.6   Solve word problems that involve linear equations, formulas, and inequalities.

Example: You are selling tickets for a play that cost $3 each. You want to sell at least $50 worth. Write and solve an inequality for the number of tickets you must sell.

Standard 3 – Relations and Functions

Students sketch and interpret graphs representing given situations. They understand the concept of a function and analyze the graphs of functions.

A1.3.1   Sketch a reasonable graph for a given relationship.

> Example: Sketch a reasonable graph for a person's height from age 0 to 25.

A1.3.2   Interpret a graph representing a given situation.

> Example: Jessica is riding a bicycle. The graph below shows her speed as it relates to the time she has spent riding. Describe what might have happened to account for such a graph.

A1.3.3   Understand the concept of a function, decide if a given relation is a function, and link equations to functions.

> Example: Use either paper or a spreadsheet to generate a list of values for x and y in $y = x^2$. Based on your data, make a conjecture about whether or not this relation is a function. Explain your reasoning.

A1.3.4   Find the domain and range of a relation.

> Example: Based on the list of values from the last example, what do the domain and range of $y = x^2$ appear to be? How can you decide whether you are correct?

Standard 4 – Graphing Linear Equations and Inequalities

Students graph linear equations and inequalities in two variables. They write equations of lines and find and use the slope and y-intercept of lines. They use linear equations to model real data.

A1.4.1   Graph a linear equation.

      Example: Draw the graph of the line with slope 3 and y-intercept -2.

A1.4.2   Find the slope, x-intercept and y-intercept of a line given its graph, its equation, or two points on the line.

      Example: Find the slop and y-intercept of the line $4x + 6y = 12$.

A1.4.3   Write the equation of a line in slope-intercept form. Understand how the slope and y-intercept of the graph are related to the equation.

      Example: Write the equation of the line $4x + 6y = 12$ in slope-intercept form. What is the slope of this line? Explain your answer.

A1.4.4   Write the equation of a line given appropriate information.

      Example: Find an equation of the line through the points (1,4) and (3,10), then find an equation of the line through the point (1,4) perpendicular to the first line.

A1.4.5   Write the equation of a line that models a data set and use the equation (or the graph of the equation) to make predictions. Describe the slope of the line in terms of the data, recognizing that the slope is the rate of change.

      Example: As your family is traveling along an interstate, you note the distance traveled every 5 minutes. A graph of time and distance shows that the relation is approximately linear. Write the equation of the line that fits your data. Predict the time for a journey of 50 miles. What does the slope represent?

A1.4.6   Graph a linear inequality in two variables.

      Example: Draw the graph of the inequality $6x + 8y \geq 24$.

Standard 5 – Pairs of Linear Equations and Inequalities

Students solve pairs of linear equations using graphs and using algebra. They solve pairs of linear inequalities using graphs. They solve word problems involving pairs of linear equations.

A1.5.1   Use a graph to estimate the solution of a pair of linear equations in two variables.

Example: Graph the equations $3y - x = 0$ and $2x + 4y = 15$ to find where the lines intersect.

A1.5.2   Use a graph to find the solution set of a pair of linear inequalities in two variables.

Example: Graph the inequalities $y \leq 4$ and $x + y \leq 5$. Shade the region where both inequalities are true.

A1.5.3   Understand and use the substitution method to solve a pair of linear equations in two variables.

Example: Solve the equations $y = 2x$ and $2x + 3y = 12$ by substitution.

A1.5.4   Understand and use the addition or subtraction method to solve a pair of linear equations in two variables.

Example: Use subtraction to solve the equations: $3x + 4y = 11$, $3x + 2y = 7$.

A1.5.5   Understand and use multiplication with the addition or subtraction method to solve a pair of linear equations in two variables.

Example: Use multiplication with the subtraction method to solve the equations: $x + 4y = 16$, $3x + 2y = -3$.

A1.5.6   Use pairs of linear equations to solve word problems.

Example: The income a company makes from a certain product can be represented by the equation $y = 10.5x$ and the expenses for that product can be represented by the equation $y = 5.25x + 10,000$, where x is the amount of the product sold and y is the number of dollars. How much of the product must be sold for the company to reach the break-even point?

Standard 6 – Polynomials

Students add, subtract, multiply, and divide polynomials. They factor quadratics.

A1.6.1   Add and subtract polynomials.

Example: Simplify $(4x^2 - 7x + 2) = (x^2 + 4x - 5)$.

A1.6.2   Multiply and divide monomials.

Example: Simplify $a^2b^5 \div ab^2$.

A1.6.3   Find powers and roots of monomials (only when the answer has an integer exponent).

Example: Find the square root of a^2b^6.

A1.6.4   Multiply polynomials.

Example: Multiply $(n + 2)(4n - 5)$.

A1.6.5   Divide polynomials by monomials.

Example: Divide $4x^3y^2 + 8xy^4 - 6x^2y^5$ by $2xy^2$.

A1.6.6   Find a common monomial factor in a polynomial.

Example: Factor $36xy^2 + 18xy^4 - 12x^2y^2$.

A1.6.7   Factor the difference of two squares and other quadratics.

Example: Factor $4x^2 - 25$ and $2x^2 - 7x + 3$.

A1.6.8   Understand and describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial expression.

Example: A graphing calculator can be used to solve $3x^2 - 5x - 1 = 0$ to the nearest tenth. Justify using the x-intercepts of y = $3x^2 - 5x - 1$ as the solutions of the equation.

Standard 7 - Algebraic Fractions

Students simplify algebraic ratios and solve algebraic proportions.

A1.7.1   Simplify algebraic ratios.

Example: Simplify $\dfrac{x^2 - 16}{x^2 + 4x}$

A1.7.2   Solve algebraic proportions.

Example: Create a tutorial to be posted to the school's web site to instruct beginning students in the steps involved in solving an algebraic proportion. Use $\frac{x+5}{4} = \frac{3x+5}{7}$ as an example.

Standard 8 - Quadratic, Cubic, and Radical Equations

Students graph and solve quadratic and radical equations. They graph cubic equations.

A1.8.1   Graph quadratic, cubic, and radical equations.

Example: Draw the graph of $y = x^2 - 3x + 2$. Using a graphing calculator or a spreadsheet (generate a data set), display the graph to check your work.

A1.8.2   Solve quadratic equations by factoring.

Example: Solve the equation $x^2 - 3x + 2 = 0$ by factoring.

A1.8.3   Solve quadratic equations in which a perfect square equals a constant.

Example: Solve the equation $(x - 7)^2 = 64$.

A1.8.4   Complete the square to solve quadratic equations.

Example: Solve the equation $x^2 - 7x + 9 = 0$ by completing the square.

A1.8.5   Derive the quadratic formula by completing the square.

Example: Prove that the equation $ax^2 + bx + c = 0$ has solutions $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

A1.8.6   Solve quadratic equations by using the quadratic formula.

Example: Solve the equation $x^2 - 7x + 9 = 0$.

A1.8.7   Use quadratic equations to solve word problems.

Example: A ball falls so that its distance above the ground can be modeled by the equation $s = 100 - 16t^2$, where s is the distance above the ground in feet and t is the time in seconds. According to this model, at what time does the ball hit the ground?

A1.8.8   Solve equations that contain radical expressions.

Example: Solve the equation $\sqrt{x + 6} = x$.

A1.8.9   Use graphing technology to find approximate solutions of quadratic and cubic equations.

Example: Use a graphing calculator to solve $3x^2 - 5x - 1 = 0$ to the nearest tenth.

Standard 9 - Mathematical Reasoning and Problem Solving

Students use a variety of strategies to solve problems.

A1.9.1   Use a variety of problem solving strategies, such as drawing a diagram, making a chart, guess-and-check, solving a simpler problem, writing an equation, and working backwards.

> Example: Fran has scored 16, 23, and 30 points in her last three games. How many points must she score in the next game so that her four game average does not fall below 20 points?

A1.9.2   Decide whether a solution is reasonable in the context of the original situation.

> Example: John says the answer to the problem in the first example is 10 points. Is his answer reasonable? Why or why not?

Students develop and evaluate mathematical arguments and proofs.

A1.9.3   Use the properties of the real number system and the order of operations to justify the steps of simplifying functions and solving equations.

> Example: Given an argument (such as $3x + 7 > 5x + 1$, and therefore $-2x > -6$, and therefore $x > 3$), provide a visual presentation of a step-by-step check, highlighting any errors in the argument.

A1.9.4   Understand that the logic of equation solving begins with the assumption that the variable is a number that satisfies the equation, and that the steps taken when solving equations create new equations that have, in most cases, the same solution set as the original. Understand that similar logic applies to solving systems of equations simultaneously.

> Example: Try "solving" the equations $x + 3y = 5$ and $5x + 15y = 25$ simultaneously, and explain what went wrong.

A1.9.5   Decide whether a given algebraic statement is true always, sometimes, or never (statements involving linear or quadratic expressions, equations, or inequalities).

> Example: Is the statement $x^2 - 5x + 2 = x^2 + 5x + 2$ true always, sometimes, or never? Explain your answer.

A1.9.6   Distinguish between inductive and deductive reasoning, identifying and providing examples of each.

> Example: What type of reasoning are you using when you look for a pattern?

AI.9.7   Identify the hypothesis and conclusion in a logical deduction.

> Example: What is the hypothesis and conclusion in this argument: If there is a numbers such that $2x + 1 = 7$, then $x = 3$.

A1.9.8   Use counter

examples to show that statements are false, recognizing that a single counterexample is sufficient to prove a general statement false.

> Example: Use the demonstration-graphing calculator on an overhead projector to produce an example showing that this statement is false: all quadratic equations have two different solutions.